Zadanie 3. (6 pkt)

Wyznacz wszystkie wartości parametru m, dla których równanie $(m^2-m)x^2-x+1=0$ ma dwa różne rozwiązania rzeczywiste x_1, x_2 takie, że $\frac{1}{x_1+x_2}\le\frac{m}{3}\le\frac{1}{x_1}+\frac{1}{x_2}$.

ROZWIĄZANIE:

$$\begin{cases} 1^\circ\ m^2-m\neq 0 \\ 2^\circ\ \ \Delta>0 \\ 3^\circ\ \frac{1}{x_1+x_2}\le\frac{m}{3}\le\frac{1}{x_1}+\frac{1}{x_2} \end{cases}$$

$1^\circ\ \ m(m-1)\neq 0$

$m\neq 0, m\neq 1$

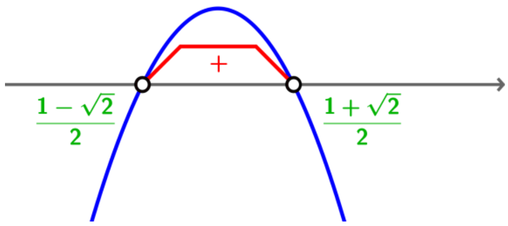
$2^\circ\ \ \Delta=1-4(m^2-m)=1-4m^2+4m=-4m^2+4m+1$

$-4m^2+4m+1>0$

$\Delta_m=16+16=32$

$\sqrt{\Delta_m}=\sqrt{32}=4\sqrt{2}$

$m_{1,2}=\frac{-4\pm4\sqrt{2}}{-8}=\frac{1\pm\sqrt{2}}{2}$



$m\in\left(\frac{1-\sqrt{2}}{2};\frac{1+\sqrt{2}}{2}\right)$

3° $\frac{1}{x_1+x_2} \leq \frac{m}{3}$ i $\frac{m}{3} \leq \frac{1}{x_1} + \frac{1}{x_2}$

$\frac{1}{\frac{1}{m^2-m}} \leq \frac{m}{3}$

$m^2 - m \leq \frac{m}{3} \quad | \cdot 3$

$3m^2 - 4m \leq 0$

$m(3m-4) \leq 0$

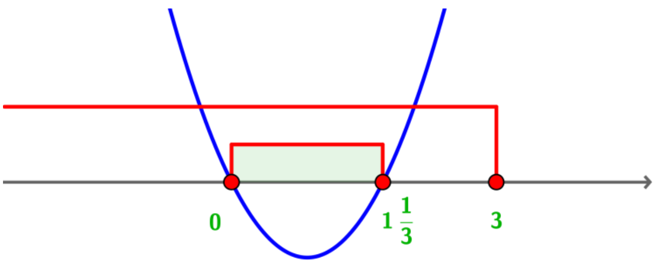
$m = 0 \qquad m = \frac{4}{3} = 1\frac{1}{3}$

$\frac{m}{3} \leq \frac{x_2 + x_1}{x_1 x_2}$

$\frac{m}{3} \leq \frac{\frac{1}{m^2-m}}{\frac{1}{m^2-m}}$

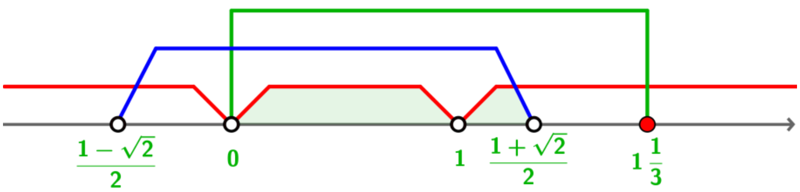
$\frac{m}{3} \leq 1 \quad | \cdot 3$

$m \leq 3$



$m \in \langle 0; 1\frac{1}{3} \rangle$

1° ∩ 2° ∩ 3°



$m \in (0;1) \cup \left(1; \frac{1+\sqrt{2}}{2}\right)$

Odp. $m \in (0;1) \cup \left(1; \frac{1+\sqrt{2}}{2}\right)$