

Zadanie 3. (6 pkt)

Wyznacz wszystkie wartości parametru m , dla których równanie $(m^2 - m)x^2 - x + 1 = 0$ ma dwa różne rozwiązania

rzeczywiste x_1, x_2 takie, że $\frac{1}{x_1 + x_2} \leq \frac{m}{3} \leq \frac{1}{x_1} + \frac{1}{x_2}$.

ROZWIĄZANIE:

$$\begin{cases} 1^\circ m^2 - m \neq 0 \\ 2^\circ \Delta > 0 \\ 3^\circ \frac{1}{x_1 + x_2} \leq \frac{m}{3} \leq \frac{1}{x_1} + \frac{1}{x_2} \end{cases}$$

$$\begin{aligned} 1^\circ m(m - 1) &\neq 0 \\ m &\neq 0, m \neq 1 \end{aligned}$$

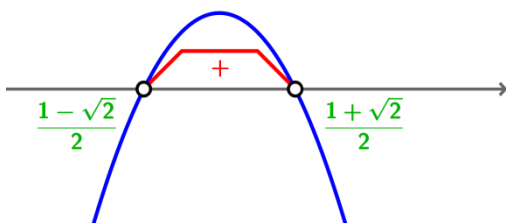
$$2^\circ \Delta = 1 - 4(m^2 - m) = 1 - 4m^2 + 4m = -4m^2 + 4m + 1$$

$$-4m^2 + 4m + 1 > 0$$

$$\Delta_m = 16 + 16 = 32$$

$$\sqrt{\Delta_m} = \sqrt{32} = 4\sqrt{2}$$

$$m_{1,2} = \frac{-4 \pm 4\sqrt{2}}{-8} = \frac{1 \pm \sqrt{2}}{2}$$



$$m \in \left(\frac{1 - \sqrt{2}}{2}; \frac{1 + \sqrt{2}}{2} \right)$$

$$3^\circ \quad \frac{1}{x_1 + x_2} \leq \frac{m}{3} \quad \text{i} \quad \frac{m}{3} \leq \frac{1}{x_1} + \frac{1}{x_2}$$

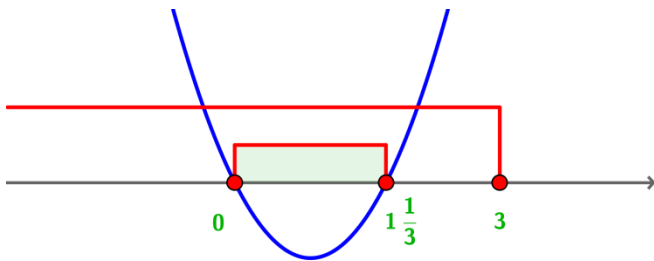
$$\frac{1}{\frac{1}{m^2 - m}} \leq \frac{m}{3} \quad \frac{m}{3} \leq \frac{x_2 + x_1}{x_1 x_2}$$

$$m^2 - m \leq \frac{m}{3} \quad | \cdot 3 \quad \frac{m}{3} \leq \frac{1}{\frac{m^2 - m}{1}}$$

$$3m^2 - 4m \leq 0 \quad \frac{m}{3} \leq 1 \quad | \cdot 3$$

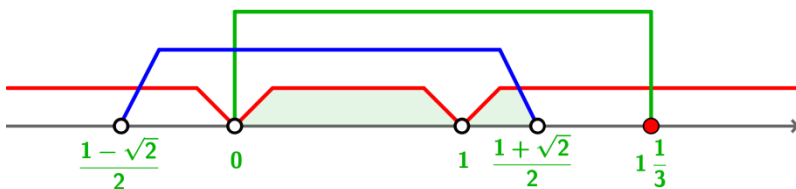
$$m(3m - 4) \leq 0 \quad m \leq 3$$

$$m = 0 \quad m = \frac{4}{3} = 1\frac{1}{3}$$



$$m \in (0; 1\frac{1}{3})$$

$$1^\circ \cap 2^\circ \cap 3^\circ$$



$$m \in (0; 1) \cup (1; \frac{1+\sqrt{2}}{2})$$

$$\text{Odp. } m \in (0; 1) \cup (1; \frac{1+\sqrt{2}}{2})$$